6.1 The Rectangular Coordinate System

In this chapter we want to take a look at the connection between algebra and geometry. This connection is made in the graphing of equations. We will start by looking at the simplest type of equation to graph, the linear equation.

Let's begin with the following definition:

Definition: **Linear equation in two variables**- an equation that can be written in the form Ax + By = C

There are a number of things we want to do with these linear equations, but one of the most important it to find **solutions** to these equations. As always, a solution to an equation is the value for the variable that makes the equation a true statement. In this case, since the equation has two variables, x and y, the solution must have both an x and y part.

We write these solutions as an **ordered pair** (x, y)

Example 1:

a. Is (-1, 3) a solution to y = 2x +4? b. Is (0, 4)?

Solution:

a. To determine if a given ordered pair is a solution to the equation, we need to find out if the ordered pair satisfies the equation. That is to say, does it make the equation a true statement when it is put into the equation. Keep in mind that ordered pairs are always written in alphabetical order.

   So, for the ordered pair (-1, 3), x = -1 and y = 3. If we put them into the equation we get

$$y = 2x + 4$$
$$3 = 2(-1) + 4$$
$$3 = -2 + 4$$
$$3 = 2$$

   Since we ended up with a statement that is clearly not true, (-1, 3) is not a solution to y = 2x +4

b. Likewise we need to plug the ordered pair into the equation and see if we get a true statement. This time, x = 0 and y = 4. We get

$$y = 2x + 4$$
$$4 = 2(0) + 4$$
$$4 = 0 + 4$$
$$4 = 4$$

   So since we ended up with a true statement, (-1, 3) is a solution to y = 2x + 4.

Even though finding out whether or not a given ordered pair is a solution, it's far more important to be able to determine the solutions for equations when we are only given the equation itself.

We can use the following process to find solutions.

Finding an ordered pair solution

1. Choose a value for one of the variables.
2. Replace the variable with its chosen value and solve the resulting equation.
3. Write the solution as an ordered pair.

Example 2:

Find the ordered pair solution for the given equation for the given value of x or y.

a. x + 4y = 0, for x = - 4, x = 0, y = - 1

b. 2x + 7y =1, for x = 3, y = - 2

Solution:

a. Let's start with the value x = - 4. Since we have been given the value of x, all we need to do is plug the value into the equation, and solve whatever we are left with as follows

$$x + 4y = 0$$

$$-4 + 4y = 0 \quad \text{Plug in -4 for x}$$

$$+4 \qquad + 4 \quad \text{Add 4 to both sides}$$

$$\frac{4y}{4} = \frac{4}{4} \quad \text{Divide by 4 to get x alone}$$

$$y = 1$$

So combining the x and y values together in an ordered pair, and keeping in mind that an ordered pair is always the x value followed by the y value, gives us the solution of (-4, 1)

For the next value, we do the same thing. Simply plug x = 0 into the equation and solve.

$$x + 4y = 0$$

$$0 + 4y = 0$$

$$\frac{4y}{4} = \frac{0}{4}$$

$$y = 0$$

So this gives the solution of (0, 0)

Lastly, we plug in the value of y = -1. This time, though, we are putting in a y value and finding an x. We get

$$x + 4y = 0$$

$$x + 4(-1) = 0$$

$$x - 4 = 0$$

$$+4 \qquad + 4$$

$$x = 4$$

This gives us the ordered pair (4, -1)

b. Just like in part a, we just need to plug each given value into the equation and solve the resulting equation. We get

For x = 3:

$$2x + 7y = 1$$

$$2(3) + 7y = 1$$

$$6 + 7y = 1$$

$$-6 \qquad - 6$$

$$\frac{7y}{7} = \frac{-5}{7}$$

$$y = -\frac{5}{7}$$

For y = -2:

$$2x + 7y = 1$$

$$2x + 7(-2) = 1$$

$$2x - 14 = 1$$

$$+14 \quad + 14$$

$$\frac{2x}{2} = \frac{15}{2}$$

$$x = \frac{15}{2}$$

So we get ordered pairs of $\left(3, -\frac{5}{7}\right)$ and $\left(\frac{15}{2}, -2\right)$.

If we are not given specific values for x and y, we can generate our own values. It turns out we can use any values we want for x or y and write them in a table. We call this table a **table of values**.

Example 3:

Generate a table of values for the equation.

a. $x - 2y = 4$

b. $y = \frac{1}{2}x - 3$

Solution:

a. In order to make a table of values, we just have to generate several ordered pair solutions for the equation. Since we are not given specific values of x and y, as we were in example 2, we get to choose the values ourself. We can, literally, choose any values we want for x and y. And, furthermore, we can choose as many values as we want. To simplify this a little, we usually like to use anywhere from 3 to 5 values, a few positives and a few negatives, and zeros are always good. The most common values are using x = -2, -1, 0, 1, 2. We put them in a table as follows

x	y
-2	
-1	
0	
1	
2	

Now we just plug each value into the equation and solve for y. The work is shown below.

For x = -2:

$$x - 2y = 4$$
$$-2 - 2y = 4$$
$$+2 \qquad +2$$
$$\frac{-2y}{-2} = \frac{6}{-2}$$
$$y = -3$$

For x = -1:

$$x - 2y = 4$$
$$-1 - 2y = 4$$
$$+1 \qquad +1$$
$$\frac{-2y}{-2} = \frac{5}{-2}$$
$$y = -\frac{5}{2}$$

For x = 0:

$$0 - 2y = 4$$
$$-2y = 4$$
$$\frac{-2y}{-2} = \frac{4}{-2}$$
$$y = -2$$

For x = 1:

$$x - 2y = 4$$
$$1 - 2y = 4$$
$$-1 \qquad -1$$
$$\frac{-2y}{-2} = \frac{3}{-2}$$
$$y = -\frac{3}{2}$$

For x = 2:

$$x - 2y = 4$$
$$2 - 2y = 4$$
$$-2 \qquad -2$$
$$\frac{-2y}{-2} = \frac{2}{-2}$$
$$y = -1$$

So filling these values in, we get the following table of values:

x	y
-2	-3
-1	$-\frac{5}{2}$
0	-2
1	$-\frac{3}{2}$
2	-1

b. Again, we can use any values that we wish to generate our table of values. So, as above, we might as well use the simple values of x = -2, -1, 0, 1, 2. However, since the equation has the fraction of ½ in front of the x, it might make more sense to use values that work well with the ½ , such as, x = -4, -2, 0, 2, 4. That way, we can minimize the amount of fractions that we need to deal with.

So, plugging those values into the equation, we get

For x = -4:
$y = \frac{1}{2}x - 3$
$y = \frac{1}{2}(-4) - 3$
$y = -2 - 3$
$y = -5$

For x = -2:
$y = \frac{1}{2}x - 3$
$y = \frac{1}{2}(-2) - 3$
$y = -1 - 3$
$y = -4$

For x = 0:
$y = \frac{1}{2}x - 3$
$y = \frac{1}{2}(0) - 3$
$y = 0 - 3$
$y = -3$

For x = 2:
$y = \frac{1}{2}x - 3$
$y = \frac{1}{2}(2) - 3$
$y = 1 - 3$
$y = -2$

For x = 4:
$y = \frac{1}{2}x - 3$
$y = \frac{1}{2}(4) - 3$
$y = 2 - 3$
$y = -1$

This gives us the table of values of

x	y
-4	-5
-2	-4
0	-3
2	-2
4	-1

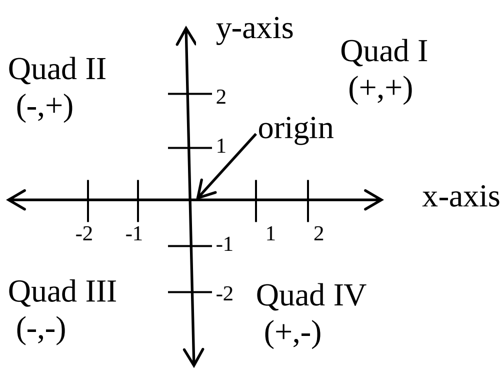
Now that we can find ordered pair solutions, what do we do with them?

Furthermore, since we can choose whatever values we want for x and y, we have an infinite number of possible ordered pair solutions.

For this reason, we want to be able to visualize this infinite collection of order pair solutions. The best way to do that is to put the ordered pairs on something called the **Rectangular Coordinate System**, or Cartesian Plane.

Here is the idea

Rectangular Coordinate System



Points on this grid are always (x, y)

First, the horizontal and vertical lines are called the axis'. The x-axis is just like our old fashioned number line, positives to the right, negatives to the left. The values on that lie above the x-axis, are have positive y-values.

These axis' split the grid into 4 regions called quadrants. We number them with roman numerals as seen above. So we can see (above) that each quadrant can be characterized by its ordered pairs signs.

Taking an ordered pair and graphing it on the Rectangular Coordinate system is called **plotting** a point.

<u>Example 4</u>:

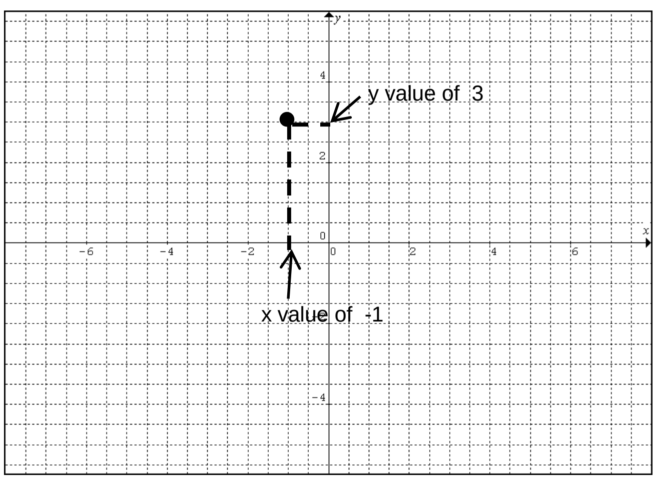
Plot the following points.

(-1, 3)	(4, 2)	(-3, -2)	(5, -4)
(1, 1)	(0, -1/2)	(3/2, 0)	

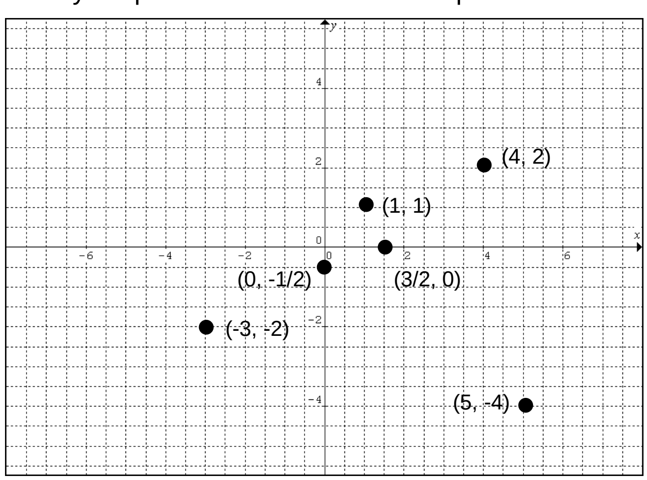
Solution:

To plot the points we need to keep in mind that the ordered pairs are always written as (x, y). So, what we do is find the point on the Rectangular Coordinate System where the value from the x part of the ordered pair, meets the value from the y part of the ordered pair.

So for (-1, 3), we will go on the x-axis to -1 and on the y-axis to 3 and where those values meet, we put a point. It looks like the following



In the same way we plot the rest of our ordered pairs as seen below



Now that we can generate ordered pair solutions, and we can plot those ordered pairs, we can put these concepts together to graph our linear equations.

<u>Example 5</u>:

Graph the following.

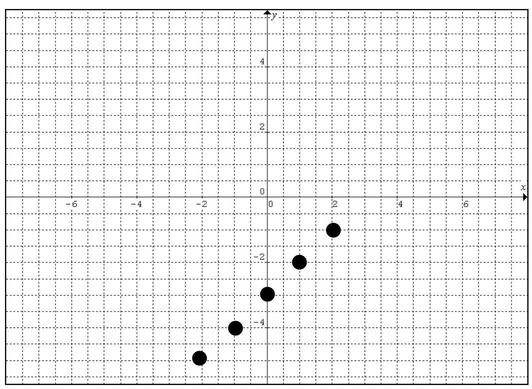
a. $y = x - 3$ b. $y = -\frac{2}{3}x + 2$ c. $4x - 2y = 2$

Solution:

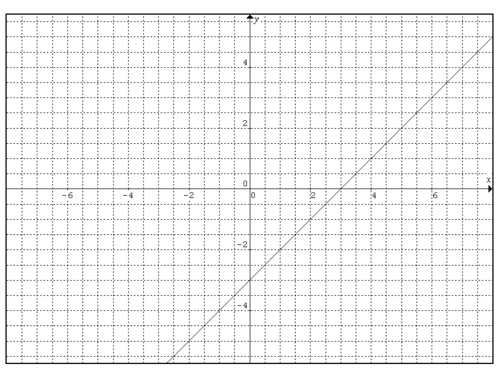
a. The first thing we want to do is generate a table of values for our equation. Again, we can use whatever values we want, however, for $y = x - 3$ our standard table of values should work fine. So we have

x	y	
-2	-5	$y = -2 - 3 = -5$
-1	-4	$y = -1 - 3 = -4$
0	-3	$y = 0 - 3 = -3$
1	-2	$y = 1 - 3 = -2$
2	-1	$y = 2 - 3 = -1$

Now we simply plot the points to see what we get. It looks like



Notice that all of the points line up. So clearly we can connect them with a line. It turns out all linear equations have graphs that are a line. That is why they are called "linear".

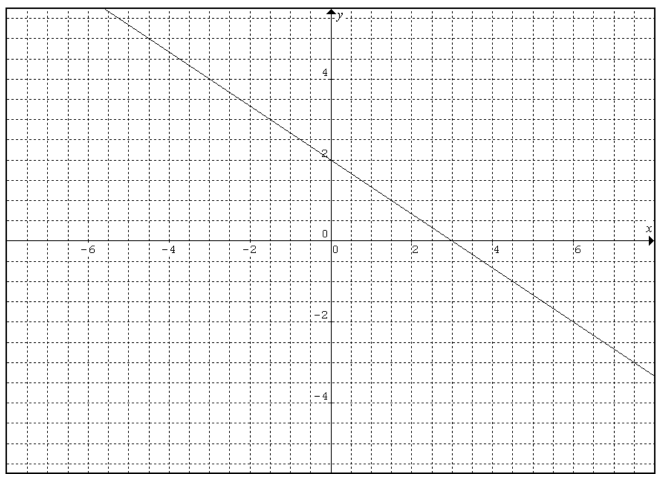


b. We graph $y = -\frac{2}{3}x + 2$ the same way. We start with the table of values and plot the points, then connect the points with a line. The only difference here is that we should use values of -6, -3, 0, 3, and 6 since those values make more sense in light of the fraction involved in the equation. We get

x	y
-6	6
-3	4
0	2
3	0
6	-2

$$y = -\frac{2}{3}(-6) + 2 = 4 + 2 = 6$$
$$y = -\frac{2}{3}(-3) + 2 = 2 + 2 = 4$$
$$y = -\frac{2}{3}(0) + 2 = 0 + 2 = 2$$
$$y = -\frac{2}{3}(3) + 2 = -2 + 2 = 0$$
$$y = -\frac{2}{3}(6) + 2 = -4 + 2 = -2$$

Plotting gives



c. Lastly we do the same for 4x – 2y = 2. The difference here is only that producing the table of values is more difficult.

x	y
-2	-5
-1	-3
0	-1
1	1
2	3

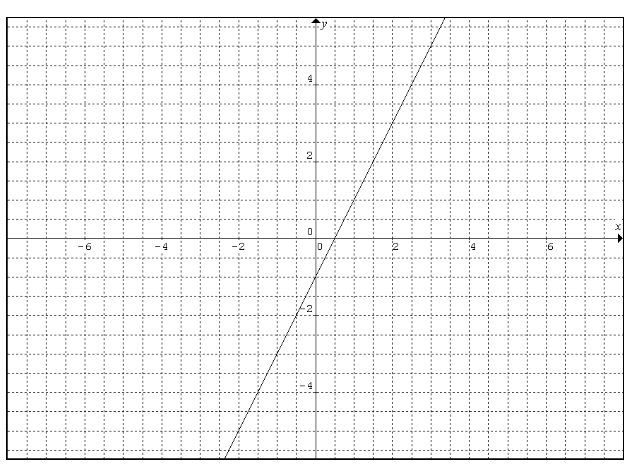
$$\begin{aligned} 4(-2) - 2y &= 2 \\ -8 - 2y &= 2 \\ -2y &= 10 \\ y &= -5 \end{aligned}$$

$$\begin{aligned} 4(-1) - 2y &= 2 \\ -4 - 2y &= 2 \\ -2y &= 6 \\ y &= -3 \end{aligned}$$

$$\begin{aligned} 4(0) - 2y &= 2 \\ 0 - 2y &= 2 \\ -2y &= 2 \\ y &= -1 \end{aligned}$$

$$\begin{aligned} 4(1) - 2y &= 2 \\ 4 - 2y &= 2 \\ -2y &= -2 \\ y &= 1 \end{aligned}$$

$$\begin{aligned} 4(2) - 2y &= 2 \\ 8 - 2y &= 2 \\ -2y &= -6 \\ y &= 3 \end{aligned}$$



6.1 Exercises

1. Is $(-3,4)$ a solution to the equation $3x+2y=-1$?
2. Is $(-2,1)$ a solution to the equation $5x+y=9$?
3. Is $(1,4)$ a solution to the equation $2x-4y=5$?
4. Is $(3,-4)$ a solution to the equation $4x+2y=4$?
5. Is $(3,5)$ a solution to the equation $y=-\frac{2}{3}x+7$?
6. Is $(-3,1)$ a solution to the equation $y=\frac{4}{5}x+\frac{7}{5}$?

Find the value that makes the ordered pair a solution to the given equation.

7. $y=\frac{2}{3}x-2$, $(3,a)$
8. $y=\frac{5}{2}x+1$, $(-2,b)$
9. $3x+4y=2$, $(c,1)$
10. $2x-3y=4$, $(d,-2)$

Find the ordered pair solution for the given equation for the given value of x or y.

11. $y=2x-3$, $x=-1$
12. $y=\frac{1}{2}x+2$, $x=4$
13. $3x-2y=4$, $y=4$
14. $4x+3y=12$, $y=-8$
15. $y=-\frac{2}{3}x+5$, $y=6$
16. $y=\frac{5}{4}x-2$, $y=3$
17. $5x-6y=12$, $x=-2$
18. $3x+4y=-12$, $x=4$

Complete the table of values for each given equation.

19. $y=-\frac{1}{3}x+2$.

x	y
-6	
	2
3	
	0

20. $y=\frac{3}{4}x-1$.

x	y
4	
	-4
0	
$\frac{4}{3}$	

21. $2x-3y=6$.

x	y
-3	
	4
0	
	0

22. $4x+7y=28$.

x	y
7	
	4
-7	
	-4

23. Plot the following points on a rectangular coordinate system.

$$A(3,-2),\ B(4,3),\ C(-7,0),\ D\left(-\frac{5}{2},-2\right)$$

24. Plot the following points on a rectangular coordinate system.

$$A(-2,-1),\ B(0,3),\ C(-3,2),\ D(3,-1)$$

25. Plot the following points on a rectangular coordinate system.

$$A(-2,3),\ B(3,4),\ C(4,-2),\ D(-2,-5),\ E\left(\frac{1}{3},\frac{5}{3}\right)$$

26. Plot the following points on a rectangular coordinate system.

$$A(1,-2),\ B(2,0),\ C(2,4),\ D(-1,-7),\ E\left(\frac{7}{2},-\frac{1}{4}\right)$$

Graph the following equations using a table of values.

27. $y = 2x - 3$
28. $y = -3x + 4$
29. $y = -x + 2$
30. $y = 5x + 1$
31. $y = \frac{1}{2}x + 3$
32. $y = -\frac{2}{3}x - 1$
33. $y = -\frac{4}{5}x + 2$
34. $y = \frac{3}{4}x - 4$
35. $y = 1.2x + 3$
36. $y = -2.5x - 1$
37. $2x - 3y = 9$
38. $4x - y = 0$
39. $5x - 4y = 8$
40. $2x - 3y = -6$
41. $3x + 5y = -15$
42. $4x + 2y = 2$
43. $x - y = 0$
44. $x + y = 4$
45. $3x + 5y = 8$
46. $4x + 3y = 12$
47. $5x - 6y = 12$
48. $2x + 3y = 5$