

Lesson 3 – Linear Equations and Functions

The first Function that we are going to investigate is the Linear Function. This is a good place to start because with Linear Functions, the average rate of change is constant and no exponents are involved. Before we begin working with Linear Functions, though, we need to review the characteristics of Linear Equations and operations on Linear Equations.

Lesson Topics:

Section 3.1: Linear Equations and Functions

- Slope
- Slope-Intercept form of the equation of a line, $y = mx + b$
- Vertical Intercepts
- Horizontal Intercepts

Section 3.2: Graphs of Linear Functions

- Graph a linear equation by plotting points
- Use the slope to graph a linear equation
- Use the intercepts to graph a linear equation

Section 3.3: Horizontal and Vertical Lines

- Equations of horizontal and vertical lines
- Graphs of horizontal and vertical lines

Section 3.4: Writing the Equation of a Line

- Write linear equations from graphs
- Applications of linear equations

Lesson 3 Checklist

Component	Required? Y or N	Comments	Due	Score
Mini-Lesson				
Online Homework				
Online Quiz				
Online Test				
Practice Problems				
Lesson Assessment				

Mini-Lesson 3

Section 3.1 – Linear Equations and Functions

The topic of linear equations should be at least slightly familiar to students starting Intermediate Algebra. The basics are covered here with reminders of important ideas and concepts that will be heavily utilized in the next lesson.

Slope

Slope is a measure of steepness and direction for a given line. It is denoted by the letter m . Given any two points, (x_1, y_1) , (x_2, y_2) , on a line, the slope is determined by computing the following ratio:

$$m = \frac{\text{Change in Output}}{\text{Change in Input}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$$

Note: If the slope is negative, then the line decreases from left to right.

If slope is positive, then the line increases from left to right.

If the slope is zero, then the line is horizontal (constant)

A vertical line has no slope (the slope is undefined).

Problem 1 | WORKED EXAMPLE – Determine Slope of a Linear Equation / Function

Find the slope of the line through the given points, then determine if the line is increasing, decreasing, horizontal, or vertical.

a) $(2, -5)$ and $(-3, 4)$.

$$\begin{aligned} m &= \frac{4 - (-5)}{-3 - (2)} \\ &= \frac{4 + 5}{-5} \\ &= \frac{9}{-5} \\ &= -\frac{9}{5} \end{aligned}$$

Decreasing

b) $(-2, -4)$ and $(4, 8)$

$$\begin{aligned} m &= \frac{8 - (-4)}{4 - (-2)} \\ &= \frac{8 + 4}{4 + 2} \\ &= \frac{12}{6} \\ &= 2 \end{aligned}$$

Increasing

c) $(2, 5)$ and $(8, 5)$

$$\begin{aligned} m &= \frac{5 - 5}{8 - 2} \\ &= \frac{0}{6} \\ &= 0 \end{aligned}$$

Horizontal (Constant)

Problem 2 | YOU TRY – Determine Slope of a Linear Equation/Function

Find the slope of the line through the given points, then determine if the line is increasing, decreasing, horizontal, or vertical.

a) (5, -2) and (-3, 4).

B) (6, 2) and (4, -6)

SLOPE-INTERCEPT form for the equation of a line.

A LINEAR EQUATION is an equation that can be written in the form:

$$y = mx + b$$

with slope, m , and vertical intercept $(0, b)$

Using function notation, the equation of a line can be written as $f(x) = mx + b$.

Vertical Intercept $(0, b)$

The vertical intercept is the special ordered pair with coordinates $(0, b)$. The input value is 0, and the resulting output is b .

The vertical intercept is often used to help when graphing a linear equation and/or to determine the initial output value in a practical application.

There are 3 main methods for finding the vertical intercept of a linear equation/function.

Method 1: Read the value of b from $y = mx + b$ or $f(x) = mx + b$ form.

Method 2: Solve for y when $x = 0$

Method 3: Evaluate $f(0)$.

Problem 3 | **WORKED EXAMPLE – Determine Vertical Intercept for a Linear Equation**

Example 1: Find the vertical intercept for the equation $y = 2x - 5$.

This equation is written in the form $y = mx + b$. Here, $b = -5$.

Therefore, (using **Method 1**) the vertical intercept is $(0, -5)$.

Example 2: Find the vertical intercept for the equation $y = 2x - 5$.

Using **Method 2**, set x to 0 and solve for y .

$$y = 2(0) - 5$$

$$y = 0 - 5$$

$$y = -5$$

The vertical intercept is $(0, -5)$

Example 3: Find the vertical intercept of the linear function $f(x) = 2x - 5$.

In this example, use **Method 3** to evaluate $f(0)$.

$$f(0) = 2(0) - 5$$

$$= 0 - 5$$

$$= -5$$

$f(0) = -5$, therefore the vertical intercept is $(0, -5)$

Problem 4 | MEDIA EXAMPLE – Determine Slope and Vertical Intercept

Complete the problems below.

Equation	$f(x) = mx + b$ form	Slope / Behavior	Vertical Intercept
a) $y = -2x + 5$			
b) $y = 2 - x$			
c) $y = \frac{3}{4}x + 2$			
d) $y = 4x$			
e) $y = -6$			
f) $y = x$			

Horizontal Intercept $(a, 0)$

The horizontal intercept is the special ordered pair with coordinates $(a, 0)$. The value a is the input value that results in an output of 0.

The horizontal intercept is often used to help when graphing a linear equation and/or to determine the final input value in a practical application.

Problem 5 | MEDIA EXAMPLE – Find The Horizontal Intercept

For each of the following problems, determine the horizontal intercept as an ordered pair.

a) $y = -2x + 5$

b) $f(x) = 2 - x$

c) $g(x) = \frac{3}{4}x + 2$

d) $y = 4x$

e) $f(x) = -6$

f) $y = x$

Problem 6 | WORKED EXAMPLE – Find The Horizontal Intercept for a Linear Equation

Find the horizontal intercept for the equation $y = 2x - 5$.

Replace the value of y with 0 then solve for the value of x .

$$0 = 2x - 5$$

$$5 = 2x$$

$$\frac{5}{2} = x$$

The horizontal intercept is $\left(\frac{5}{2}, 0\right)$

Problem 7 | YOU TRY – Find The Horizontal Intercept for a Linear Equation/Function

Complete the table below. Write intercepts as ordered pairs.

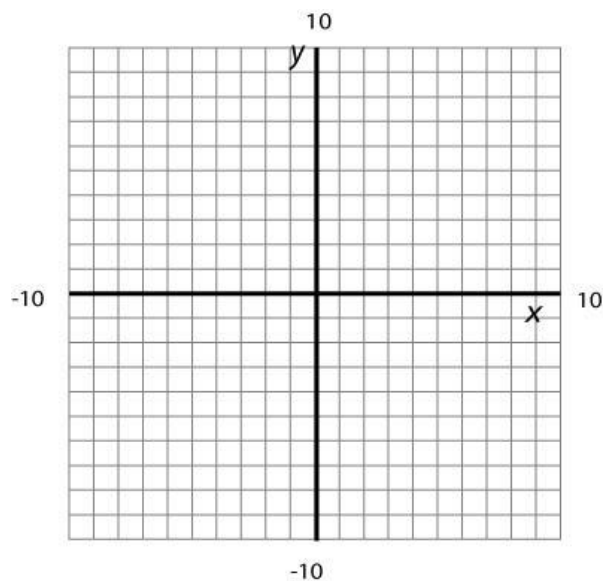
Equation	Slope / Behavior	Vertical Intercept	Horizontal Intercept
a) $f(x) = 6 - 4x$			
b) $y = 3x$			
c) $y = \frac{3}{5}x - 8$			

Section 3.2 – Graphs of Linear Functions

Problem 8 MEDIA EXAMPLE – Graphing a Linear Equation by Plotting Points

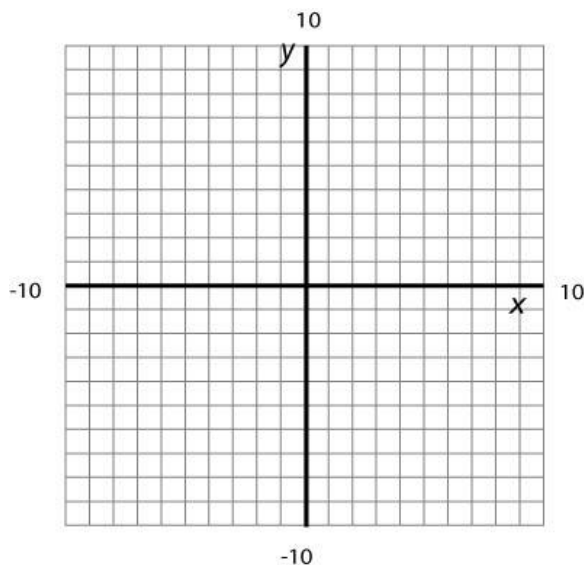
Graph the equation $f(x) = -2x + 6$

x	$f(x)$	Ordered Pair

**Problem 9** MEDIA EXAMPLE – Using the SLOPE to Graph a Linear Equation

$$SLOPE = m = \frac{\text{Change in Output}}{\text{Change in Input}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$$

Draw an accurate graph for $y = -2x + 6$. Identify at least two additional points on the line, and label them on your graph.



Problem 10 | **WORKED EXAMPLE – Use the Intercepts to Graph a Linear Equation**

Graph the equation $y = -2x + 6$ by plotting the intercepts on the graph.

Vertical Intercept: This equation is written in the form $y = mx + b$, so the vertical intercept is $(0, 6)$.

Horizontal Intercept: Set y to 0 and solve for x .

$$y = -2x + 6$$

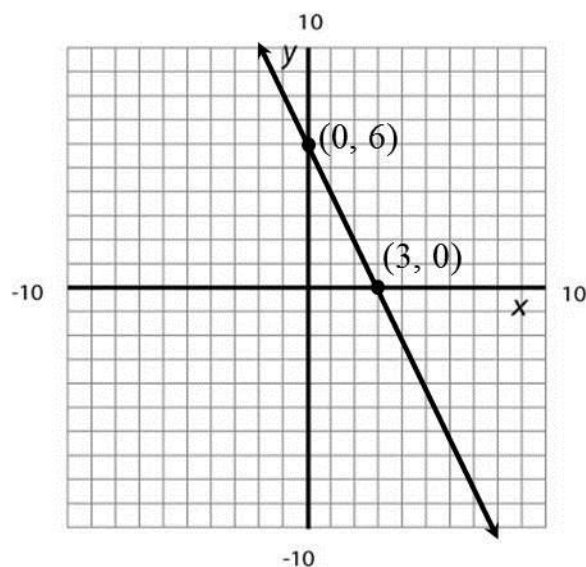
$$0 = -2x + 6$$

$$-6 = -2x$$

$$3 = x$$

So the horizontal intercept is $(3, 0)$.

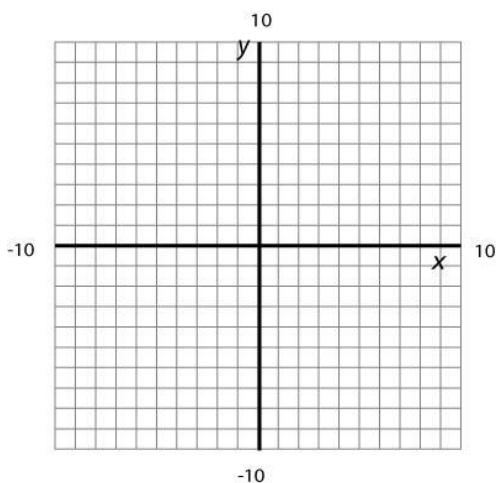
PLOT and LABEL the intercepts on the graph then connect them to draw your line.



Problem 11	YOU TRY – Draw Graphs of Linear Equations
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Use the equation $y = -\frac{3}{2}x + 6$ for all parts of this problem. Label all plotted points.

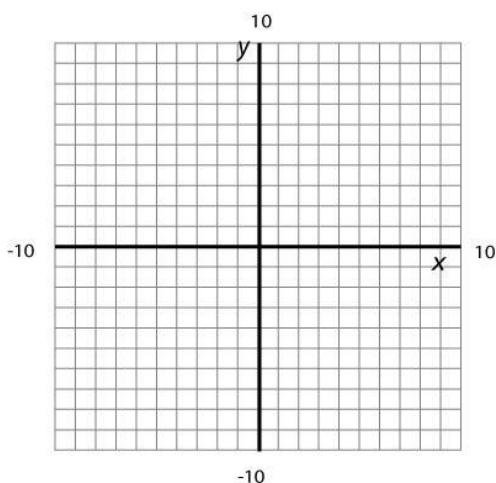
- a) Use the INTERCEPTS to draw the graph of the line. Show your work to find these points. PLOT and LABEL the intercepts on the graph then connect them to draw your line.



Vertical Intercept: (_____ , _____)

Horizontal Intercept : (_____ , _____)

- b) Use the SLOPE to graph the line. Identify at least two additional points on the line (not the intercepts), and label them on your graph.



Slope = _____

Two additional ordered pairs

(_____ , _____) (_____ , _____)

NOTICE: Your graphs for parts a) and b) should look exactly the same.

Section 3.3 – Horizontal and Vertical Lines

Vertical Lines

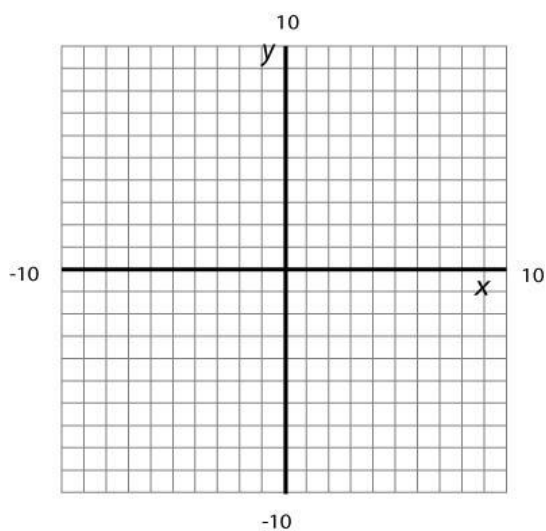
- Equation: $x = a$
- Horizontal Intercept: $(a, 0)$
- Vertical Intercept: none
- Slope: m is undefined

Horizontal Lines

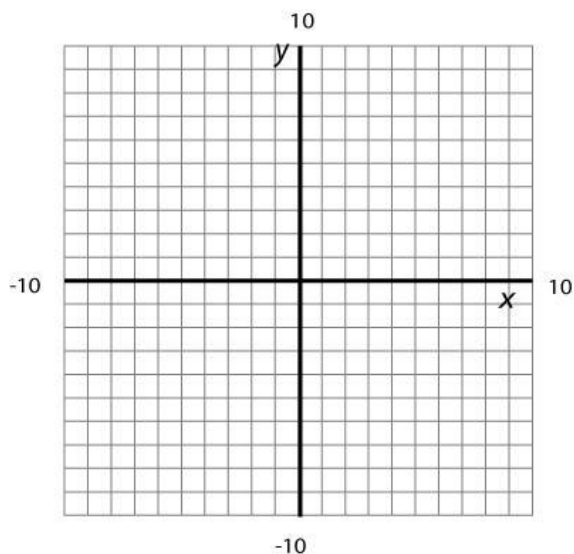
- Equation: $y = b, f(x) = b$
- Horizontal Intercept: none
- Vertical Intercept: $(0, b)$
- Slope: $m = 0$

Problem 12 **MEDIA EXAMPLE – Graphing Horizontal and Vertical Lines**

a) Use the grid below to graph the equation $y = -2$. Identify the slope and intercepts.



b) Use the grid below to graph the equations $x = 5$. Identify the slope and intercepts.



Problem 13	YOU TRY – HORIZONTAL AND VERTICAL LINES
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a) Given the ordered pair $(2, -3)$

- Sketch the graph of the vertical line through this point.
- Write the equation of the vertical line through this point. Use function notation if possible.
- Identify the slope of the line: _____
- What is the vertical intercept? _____
- What is the horizontal intercept? _____

b) Given the ordered pair $(2, -3)$

- Sketch the graph of the horizontal line through this point.
- Write the equation of the horizontal line through this point. Use function notation if possible.
- Identify the slope of the line: _____
- What is the vertical intercept? _____
- What is the horizontal intercept? _____

Section 3.4 – Writing the Equation of a Line

Writing Equations of Lines

Critical to a thorough understanding of linear equations and functions is the ability to write the equation of a line given different pieces of information. The following process will work for almost every situation you are presented with and will be illustrated several times in the media problems to follow.

Step 1: Determine the value of the slope, m .

Step 2: Determine the coordinates of one ordered pair.

Step 3: Plug the values for the ordered pair, and the value for the slope, into $y = mx + b$

Step 4: Solve for b

Step 5: Use the values for m and b to write the resulting equation in $y = mx + b$ form.

Step 6: When appropriate, rewrite the equation in function notation: $f(x) = mx + b$.

Problem 14	MEDIA EXAMPLE – Writing Equations of Lines
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For each of the following, find the equation of the line that meets the following criteria:

a) Slope $m = -4$ passing through the point $(0, 3)$.

b) Passing through the points $(0, -2)$ and $(1, 5)$

c) Passing through the points $(-2, -3)$ and $(4, -9)$

d) Parallel to $y = 3x - 7$ and passing through $(2, -5)$

e) Horizontal line passing through $(-3, 5)$.

f) Vertical line passing through $(-3, 5)$.

Problem 15 **WORKED EXAMPLE – Writing Equations of Lines**

Write an equation of the line to satisfy each set of conditions.

- a) A line that contains the points $(-3, 5)$ and $(0, 1)$

Slope: Use the ordered pairs $(-3, 5)$ and $(0, 1)$ to compute slope.

$$m = \frac{1-5}{0-(-3)} = \frac{-4}{3} = -\frac{4}{3}$$

Vertical Intercept: The vertical intercept $(0, 1)$ is given in the problem, so $b = 1$.

Equation: Plug $m = -\frac{4}{3}$ and $b = 1$ into $y = mx + b$

$$y = -\frac{4}{3}x + 1$$

$$f(x) = -\frac{4}{3}x + 1$$

- b) Line contains points $(-4, -3)$ and $(2, 6)$

Slope: Use the ordered pairs $(-4, -3)$ and $(2, 6)$ to compute slope.

$$\begin{aligned} m &= \frac{6-(-3)}{2-(-4)} \\ &= \frac{9}{6} \\ &= \frac{3}{2} \end{aligned}$$

Vertical Intercept: Because neither of the given ordered pairs is the vertical intercept, b must be computed. Pick one of the given ordered pairs. Plug m and that ordered pair into $y = mx + b$. Solve for b .

Using $(-4, -3)$

$$-3 = \frac{3}{2}(-4) + b$$

$$-3 = -6 + b$$

$$3 = b$$

Using $(2, 6)$

$$6 = \frac{3}{2}(2) + b$$

$$6 = 3 + b$$

$$3 = b$$

Equation: Plug $m = \frac{3}{2}$ and $b = 3$ into $y = mx + b$

$$y = \frac{3}{2}x + 3$$

$$f(x) = \frac{3}{2}x + 3$$

Problem 16	YOU TRY – Writing Equations of Lines
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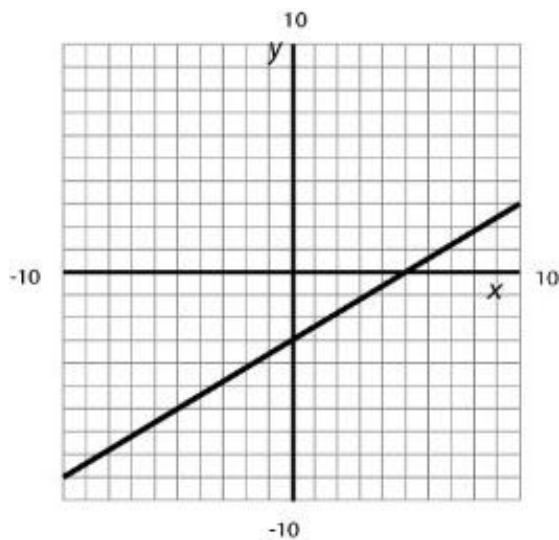
a) Find the equation of the line passing through the points (1,4) and (3, -2) and write your answer in the form $f(x) = mx + b$. Show complete work in this space.

b) What is the vertical intercept for this equation? Show work or explain your result.

c) What is the horizontal intercept for this equation? Show complete work to find this.

Problem 17 **WORKED EXAMPLE – Writing Linear Equations from Graphs**

A line has the following graph:



Slope: Identify two ordered pairs from the graph and use them to determine the slope.

$$(5, 0) \text{ and } (3, -1)$$

$$m = \frac{-1 - (0)}{3 - (5)} = \frac{-1}{-2} = \frac{1}{2}$$

Vertical intercept: Read the vertical intercept from the graph.

Ordered pair is $(0, -3)$. Therefore $b = -3$.

Equation: Plug m and b into $y = mx + b$

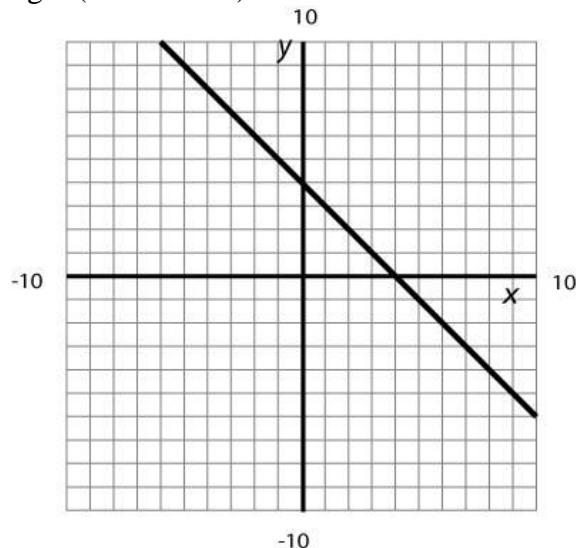
$$m = \frac{1}{2}, b = -3$$

$$y = \frac{1}{2}x - 3$$

$$f(x) = \frac{1}{2}x - 3$$

Problem 18 **YOU TRY – Writing Linear Equations from Graphs**

Use the given graph of the function f below to help answer the questions below. Assume the line intersects grid corners at integer (not decimal) values.



- Is the line above increasing, decreasing, or constant?
- What is the vertical intercept? *Also, plot and label the vertical intercept on the graph.*
- What is the horizontal intercept? *Also, plot and label the horizontal intercept on the graph.*
- What is the slope? Show your work.
- What is the equation of the line? Show complete work. Your answer must be written in function notation.

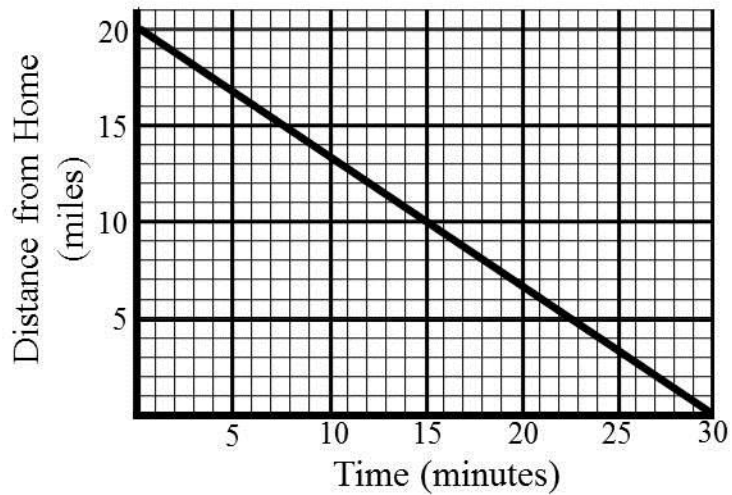
Problem 19	MEDIA EXAMPLE – Applications of Linear Functions
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A candy company has a machine that produces candy canes. The number of candy canes produced depends on the amount of time the machine has been operating. The machine produces 160 candy canes in five minutes. In twenty minutes, the machine can produce 640 candy canes.

- Determine a linear equation to model this situation. Clearly indicate what each variable represents.
- Determine the vertical intercept of this linear equation. Write it as an ordered pair and interpret its practical meaning.
- Determine the horizontal intercept of this linear equation. Write it as an ordered pair and interpret its practical meaning.
- How many candy canes will this machine produce in 1 minute?
- How many candy canes will this machine produce in 1 hour?

Problem 20 | **YOU TRY – Applications of Linear Functions**

The graph below shows a person's distance from home as a function of time.



- Identify the vertical intercept. Write it as an ordered pair and interpret its practical meaning.
- Identify the horizontal intercept. Write it as an ordered pair and interpret its practical meaning.
- Determine a linear equation to model this situation. Indicate what each variable represents.
- How far has this person traveled in one minute?

