1.4 Multiply and Divide Integers

Learning Objectives

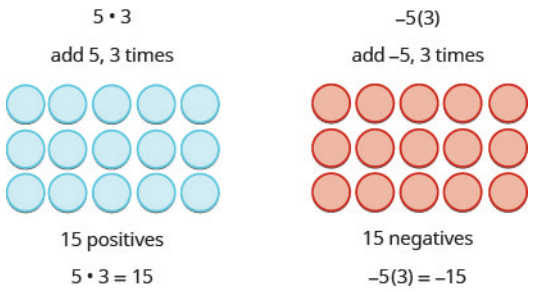
By the end of this section, you will be able to:

- Multiply integers
- Divide integers
- Simplify expressions with integers
- Evaluate variable expressions with integers
- Translate English phrases to algebraic expressions
- Use integers in applications

Multiply Integers

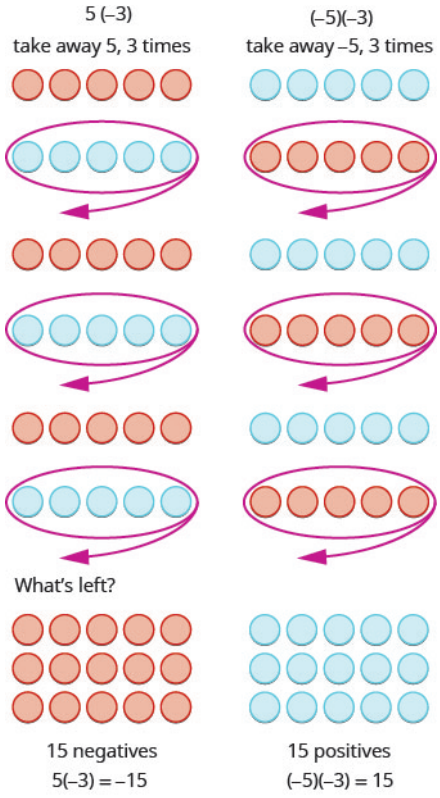
Since multiplication is mathematical shorthand for repeated addition, our model can easily be applied to show multiplication of integers. Let's look at this concrete model to see what patterns we notice. We will use the same examples that we used for addition and subtraction. Here, we will use the model just to help us discover the pattern.

We remember that $a \cdot b$ means add a, b times. Here, we are using the model just to help us discover the pattern.



The next two examples are more interesting.

What does it mean to multiply 5 by -3? It means subtract 5, 3 times. Looking at subtraction as "taking away," it means to take away 5, 3 times. But there is nothing to take away, so we start by adding neutral pairs on the workspace. Then we take away 5 three times.



In summary:

$$5 \cdot 3 = 15 \qquad -5(3) = -15$$
$$5(-3) = -15 \qquad (-5)(-3) = 15$$

Notice that for multiplication of two signed numbers, when the:

- signs are the *same*, the product is *positive*.
- signs are *different*, the product is *negative*.

We'll put this all together in the chart below.

Multiplication of Signed Numbers

For multiplication of two signed numbers:

Same signs	Product	Example
Two positives Two negatives	Positive Positive	$7 \cdot 4 = 28$ $-8(-6) = 48$

Different signs	Product	Example
Positive · negative Negative · positive	Negative Negative	$7(-9) = -63$ $-5 \cdot 10 = -50$

EXAMPLE 1.46

**Multiply: ⓐ $-9 \cdot 3$ ⓑ $-2(-5)$ ⓒ $4(-8)$ ⓓ $7 \cdot 6$.

Solution

ⓐ Multiply, noting that the signs are different so the product is negative.	$-9 \cdot 3$ -27
ⓑ Multiply, noting that the signs are the same so the product is positive.	$-2(-5)$ 10
ⓒ Multiply, with different signs.	$4(-8)$ -32
ⓓ Multiply, with same signs.	$7 \cdot 6$ 42

Table 1.8

TRY IT : : 1.91

TRY IT :: 1.91

Multiply: ⓐ $-6 \cdot 8$ ⓑ $-4(-7)$ ⓒ $9(-7)$ ⓓ $5 \cdot 12$.

TRY IT : : 1.92

TRY IT :: 1.92

Multiply: ⓐ $-8 \cdot 7$ ⓑ $-6(-9)$ ⓒ $7(-4)$ ⓓ $3 \cdot 13$.

When we multiply a number by 1, the result is the same number. What happens when we multiply a number by -1? Let's multiply a positive number and then a negative number by -1 to see what we get.

Multiply.	$-1 \cdot 4$	$-1(-3)$
	-4	3
	-4 is the opposite of 4.	3 is the opposite of -3.

Each time we multiply a number by -1, we get its opposite!

Multiplication by -1

$$-1a = -a$$

Multiplying a number by -1 gives its opposite.

EXAMPLE 1.47

**Multiply: ⓐ $-1 \cdot 7$ ⓑ $-1(-11)$.

Solution

ⓐ Multiply, noting that the signs are different so the product is negative.	$-1 \cdot 7$ -7 -7 is the opposite of 7.
ⓑ Multiply, noting that the signs are the same so the product is positive.	$-1(-11)$ 11 11 is the opposite of -11.

Table 1.9

TRY IT : : 1.93

TRY IT :: 1.93

Multiply: ⓐ $-1 \cdot 9$ ⓑ $-1 \cdot (-17)$.

TRY IT : : 1.94

TRY IT :: 1.94

Multiply: ⓐ $-1 \cdot 8$ ⓑ $-1 \cdot (-16)$.

Divide Integers

What about division? Division is the inverse operation of multiplication. So, $15 \div 3 = 5$ because $15 \cdot 3 = 5$. In words, this expression says that 15 can be divided into three groups of five each because adding five three times gives 15. Look at some examples of multiplying integers, to figure out the rules for dividing integers.

$$5 \cdot 3 = 15 \text{ so } 15 \div 3 = 5 \qquad -5(3) = -15 \text{ so } -15 \div 3 = -5$$
$$(-5)(-3) = 15 \text{ so } 15 \div (-3) = -5 \qquad 5(-3) = -15 \text{ so } -15 \div (-3) = 5$$

Division follows the same rules as multiplication!

For division of two signed numbers, when the:

- signs are the *same*, the quotient is *positive*.
- signs are *different*, the quotient is *negative*.

And remember that we can always check the answer of a division problem by multiplying.

Multiplication and Division of Signed Numbers

For multiplication and division of two signed numbers:

- If the signs are the same, the result is positive.
- If the signs are different, the result is negative.

Same signs	Result
Two positives Two negatives	Positive Positive
If the signs are the same, the result is positive.	

Different signs	Result
Positive and negative Negative and positive	Negative Negative
If the signs are different, the result is negative.	

EXAMPLE 1.48

**Divide: ⓐ $-27 \div 3$ ⓑ $-100 \div (-4)$.

Solution

ⓐ Divide. With different signs, the quotient is negative.	$-27 \div 3$ -9
ⓑ Divide. With signs that are the same, the quotient is positive.	$-100 \div (-4)$ 25

Table 1.10

TRY IT : : 1.95

TRY IT :: 1.95

Divide: ⓐ $-42 \div 6$ ⓑ $-117 \div (-3)$.

TRY IT : : 1.96

TRY IT :: 1.96

Divide: ⓐ $-63 \div 7$ ⓑ $-115 \div (-5)$.

Simplify Expressions with Integers

What happens when there are more than two numbers in an expression? The order of operations still applies when negatives are included. Remember My Dear Aunt Sally?

Let's try some examples. We'll simplify expressions that use all four operations with integers—addition, subtraction, multiplication, and division. Remember to follow the order of operations.

EXAMPLE 1.49

**Simplify: $7(-2) + 4(-7) - 6$.

Solution

	$7(-2)+4(-7)-6$
Multiply first.	$-14+(-28)-6$
Add.	$-42-6$
Subtract.	-48

Table 1.11

TRY IT : : 1.97

TRY IT :: 1.97

Simplify: $8(-3)+5(-7)-4$.

TRY IT : : 1.98

TRY IT :: 1.98

Simplify: $9(-3)+7(-8)-1$.

EXAMPLE 1.50

**Simplify: ⓐ $(-2)^4$ ⓑ -2^4.

Solution

ⓐ Write in expanded form. Multiply. Multiply. Multiply.	$(-2)^4$ $(-2)(-2)(-2)(-2)$ $4(-2)(-2)$ $-8(-2)$ 16
ⓑ Write in expanded form. We are asked to find the opposite of 2^4. Multiply. Multiply. Multiply.	-2^4 $-(2\cdot 2\cdot 2\cdot 2)$ $-(4\cdot 2\cdot 2)$ $-(8\cdot 2)$ -16

Table 1.12

Notice the difference in parts ⓐ and ⓑ. In part ⓐ, the exponent means to raise what is in the parentheses, the (-2) to the 4^{th} power. In part ⓑ, the exponent means to raise just the 2 to the 4^{th} power and then take the opposite.

TRY IT : : 1.99

TRY IT :: 1.99

Simplify: ⓐ $(-3)^4$ ⓑ -3^4.

TRY IT : : 1.100

TRY IT :: 1.100

Simplify: ⓐ $(-7)^2$ ⓑ -7^2.

The next example reminds us to simplify inside parentheses first.

EXAMPLE 1.51

**Simplify: $12 - 3(9 - 12)$.

Solution

	$12 - 3(9 - 12)$
Subtract in parentheses first.	$12 - 3(-3)$
Multiply.	$12 - (-9)$
Subtract.	21

Table 1.13

TRY IT : : 1.101

TRY IT :: 1.101

Simplify: $17 - 4(8 - 11)$.

TRY IT : : 1.102

TRY IT :: 1.102

Simplify: $16 - 6(7 - 13)$.

EXAMPLE 1.52

**Simplify: $8(-9) \div (-2)^3$.

Solution

	$8(-9) \div (-2)^3$
Exponents first.	$8(-9) \div (-8)$
Multiply.	$-72 \div (-8)$
Divide.	9

Table 1.14

TRY IT : : 1.103

TRY IT :: 1.103

Simplify: $12(-9) \div (-3)^3$.

TRY IT : : 1.104

TRY IT :: 1.104

Simplify: $18(-4) \div (-2)^3$.

EXAMPLE 1.53

**Simplify: $-30 \div 2 + (-3)(-7)$.

Solution

	$-30 \div 2 + (-3)(-7)$
Multiply and divide left to right, so divide first.	$-15 + (-3)(-7)$
Multiply.	$-15 + 21$
Add.	6

Table 1.15

TRY IT : : 1.105

TRY IT :: 1.105

Simplify: $-27 \div 3 + (-5)(-6)$.

TRY IT : : 1.106

TRY IT :: 1.106

Simplify: $-32 \div 4 + (-2)(-7)$.

Evaluate Variable Expressions with Integers

Remember that to evaluate an expression means to substitute a number for the variable in the expression. Now we can use negative numbers as well as positive numbers.

EXAMPLE 1.54

**When $n = -5$, evaluate: ⓐ $n + 1$ ⓑ $-n + 1$.

Solution

ⓐ

	$n + 1$
Substitute –5 for n.	$-5 + 1$
Simplify.	-4

ⓑ

	$-n+1$
Substitute –5 for n.	$-(-5)+1$
Simplify.	$5+1$
Add.	6

TRY IT : : 1.107

TRY IT :: 1.107

When $n=-8$, evaluate ⓐ $n+2$ ⓑ $-n+2$.

TRY IT : : 1.108

TRY IT :: 1.108

When $y=-9$, evaluate ⓐ $y+8$ ⓑ $-y+8$.

EXAMPLE 1.55

**Evaluate $(x+y)^2$ when $x=-18$ and $y=24$.

Solution

	$(x+y)^2$
Substitute –18 for x and 24 for y.	56.3
Add inside parenthesis.	$(6)^2$
Simplify.	36

TRY IT : : 1.109

TRY IT :: 1.109

Evaluate $(x+y)^2$ when $x=-15$ and $y=29$.

TRY IT : : 1.110

TRY IT :: 1.110

Evaluate $(x+y)^3$ when $x=-8$ and $y=10$.

EXAMPLE 1.56

**Evaluate $20-z$ when ⓐ $z=12$ and ⓑ $z=-12$.

Solution

ⓐ

	$20-z$
Substitute 12 for z.	$20-12$
Subtract.	8

ⓑ

	$20 - z$
Substitute -12 for z.	$20 - (-12)$
Subtract.	32

TRY IT : : 1.111

TRY IT :: 1.111

Evaluate: $17 - k$ when ⓐ $k = 19$ and ⓑ $k = -19$.

TRY IT : : 1.112

TRY IT :: 1.112

Evaluate: $-5 - b$ when ⓐ $b = 14$ and ⓑ $b = -14$.

EXAMPLE 1.57

**Evaluate: $2x^2 + 3x + 8$ when $x = 4$.

Solution

Substitute 4 for x. Use parentheses to show multiplication.

	$2x^2 + 3x + 8$
Substitute.	$2(4)^2 + 3(4) + 8$
Evaluate exponents.	$2(16) + 3(4) + 8$
Multiply.	$32 + 12 + 8$
Add.	52

TRY IT : : 1.113

TRY IT :: 1.1113

Evaluate: $3x^2 - 2x + 6$ when $x = -3$.

TRY IT : : 1.114

TRY IT :: 1.114

Evaluate: $4x^2 - x - 5$ when $x = -2$.

Translate Phrases to Expressions with Integers

Our earlier work translating English to algebra also applies to phrases that include both positive and negative numbers.

EXAMPLE 1.58

**Translate and simplify: the sum of 8 and -12, increased by 3.

Solution

	the **sum** of 8 and -12, increased by 3.
Translate.	$[8+(-12)]+3$
Simplify. Be careful not to confuse the brackets with an absolute value sign.	$(-4)+3$
Add.	-1

Table 1.16

TRY IT : : 1.115

TRY IT :: 1.115

Translate and simplify the sum of 9 and -16, increased by 4.

TRY IT : : 1.116

TRY IT :: 1.116

Translate and simplify the sum of -8 and -12, increased by 7.

When we first introduced the operation symbols, we saw that the expression may be read in several ways. They are listed in the chart below.

$a-b$
a minus b
the difference of a and b
b subtracted from a
b less than a

Be careful to get *a* and *b* in the right order!

EXAMPLE 1.59

**Translate and then simplify ⓐ the difference of 13 and -21 ⓑ subtract 24 from -19.

Solution

ⓐ Translate. Simplify.	the **difference** *of* 13 *and* -21 $13-(-21)$ 34
ⓑ Translate. Remember, "subtract b from a means $a-b$. Simplify.	**subtract** 24 **from** -19 $-19-24$ 43

Table 1.17

TRY IT : : 1.117

TRY IT :: 1.117

Translate and simplify ⓐ the difference of 14 and -23 ⓑ subtract 21 from -17.

TRY IT : : 1.118

TRY IT :: 1.118

Translate and simplify ⓐ the difference of 11 and -19 ⓑ subtract 18 from -11.

Once again, our prior work translating English to algebra transfers to phrases that include both multiplying and dividing integers. Remember that the key word for multiplication is " product" and for division is " quotient."

EXAMPLE 1.60

**Translate to an algebraic expression and simplify if possible: the product of -2 and 14.

Solution

	the product *of* -2 *and* 14
Translate.	$(-2)(14)$
Simplify.	-28

Table 1.18

TRY IT : : 1.119

TRY IT :: 1.119

Translate to an algebraic expression and simplify if possible: the product of -5 and 12.

TRY IT : : 1.120

TRY IT :: 1.120

Translate to an algebraic expression and simplify if possible: the product of 8 and -13.

EXAMPLE 1.61

**Translate to an algebraic expression and simplify if possible: the quotient of -56 and -7.

Solution

	the quotient *of* -56 *and* -7
Translate.	$-56 \div (-7)$
Simplify.	8

Table 1.19

TRY IT : : 1.121

TRY IT :: 1.121

Translate to an algebraic expression and simplify if possible: the quotient of -63 and -9.

TRY IT : : 1.122

TRY IT :: 1.122

Translate to an algebraic expression and simplify if possible: the quotient of -72 and -9.

Use Integers in Applications

We'll outline a plan to solve applications. It's hard to find something if we don't know what we're looking for or what to call it! So when we solve an application, we first need to determine what the problem is asking us to find. Then we'll write a phrase that gives the information to find it. We'll translate the phrase into an expression and then simplify the expression to get the answer. Finally, we summarize the answer in a sentence to make sure it makes sense.

EXAMPLE 1.62 HOW TO APPLY A STRATEGY TO SOLVE APPLICATIONS WITH INTEGERS

**The temperature in Urbana, Illinois one morning was 11 degrees. By mid-afternoon, the temperature had dropped to -9 degrees. What was the difference of the morning and afternoon temperatures?

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.	
Step 2. Identify what we are asked to find.	the difference of the morning and afternoon temperatures
Step 3. Write a phrase the gives the information to find it.	the *difference of* 11 *and* –9
Step 4. Translate the phrase to an expression.	11 – (–9)
Step 5. Simplify the expression.	20
Step 6. Write a complete sentence that answers the question.	The difference in temperatures was 20 degrees.

TRY IT : : 1.123

TRY IT :: 1.123

The temperature in Anchorage, Alaska one morning was 15 degrees. By mid-afternoon the temperature had dropped to 30 degrees below zero. What was the difference in the morning and afternoon temperatures?

TRY IT : : 1.124

TRY IT :: 1.124

The temperature in Denver was -6 degrees at lunchtime. By sunset the temperature had dropped to -15 degrees. What was the difference in the lunchtime and sunset temperatures?

HOW TO : : APPLY A STRATEGY TO SOLVE APPLICATIONS WITH INTEGERS.

Step 1. Read the problem. Make sure all the words and ideas are understood
Step 2. Identify what we are asked to find.
Step 3. Write a phrase that gives the information to find it.
Step 4. Translate the phrase to an expression.
Step 5. Simplify the expression.
Step 6. Answer the question with a complete sentence.

EXAMPLE 1.63

**The Mustangs football team received three penalties in the third quarter. Each penalty gave them a loss of fifteen yards. What is the number of yards lost?

Solution

Step 1. Read the problem. Make sure all the words and ideas are understood.	
Step 2. Identify what we are asked to find.	the number of yards lost
Step 3. Write a phrase that gives the information to find it.	three times a 15-yard penalty
Step 4. Translate the phrase to an expression.	$3(-15)$
Step 5. Simplify the expression.	-45
Step 6. Answer the question with a complete sentence.	The team lost 45 yards.

Table 1.20

TRY IT : : 1.125

TRY IT :: 1.125

The Bears played poorly and had seven penalties in the game. Each penalty resulted in a loss of 15 yards. What is the number of yards lost due to penalties?

TRY IT : : 1.126

TRY IT :: 1.126

Bill uses the ATM on campus because it is convenient. However, each time he uses it he is charged a $2 fee. Last month he used the ATM eight times. How much was his total fee for using the ATM?

Key Concepts

- **Multiplication and Division of Two Signed Numbers**
  - Same signs—Product is positive
  - Different signs—Product is negative
- **Strategy for Applications**

  Step 1. Identify what you are asked to find.
  Step 2. Write a phrase that gives the information to find it.
  Step 3. Translate the phrase to an expression.

Step 4. Simplify the expression.

Step 5. Answer the question with a complete sentence.

1.4 EXERCISES

Practice Makes Perfect

Multiply Integers

In the following exercises, multiply.

265. $-4 \cdot 8$

266. $-3 \cdot 9$

267. $9(-7)$

268. $13(-5)$

269. -1.6

270. -1.3

271. $-1(-14)$

272. $-1(-19)$

Divide Integers

In the following exercises, divide.

273. $-24 \div 6$

274. $35 \div (-7)$

275. $-52 \div (-4)$

276. $-84 \div (-6)$

277. $-180 \div 15$

278. $-192 \div 12$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

279. $5(-6) + 7(-2) - 3$

280. $8(-4) + 5(-4) - 6$

281. $(-2)^6$

282. $(-3)^5$

283. -4^2

284. -6^2

285. $-3(-5)(6)$

286. $-4(-6)(3)$

287. $(8 - 11)(9 - 12)$

288. $(6 - 11)(8 - 13)$

289. $26 - 3(2 - 7)$

290. $23 - 2(4 - 6)$

291. $65 \div (-5) + (-28) \div (-7)$

292. $52 \div (-4) + (-32) \div (-8)$

293. $9 - 2[3 - 8(-2)]$

294. $11 - 3[7 - 4(-2)]$

295. $(-3)^2 - 24 \div (8 - 2)$

296. $(-4)^2 - 32 \div (12 - 4)$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

297. $y + (-14)$ when

ⓐ $y = -33$

ⓑ $y = 30$

298. $x + (-21)$ when

ⓐ $x = -27$

ⓑ $x = 44$

299.

ⓐ $a + 3$ when $a = -7$

ⓑ $-a + 3$ when $a = -7$

300.

ⓐ $d + (-9)$ when $d = -8$

ⓑ $-d + (-9)$ when $d = -8$

301. $m + n$ when $m = -15, n = 7$

302. $p + q$ when $p = -9, q = 17$

303. $r + s$ when $r = -9, s = -7$

304. $t + u$ when $t = -6, u = -5$

305. $(x + y)^2$ when $x = -3, y = 14$

306. $(y+z)^2$ when $y=-3,\ z=15$

307. $-2x+17$ when
ⓐ $x=8$
ⓑ $x=-8$

308. $-5y+14$ when
ⓐ $y=9$
ⓑ $y=-9$

309. $10-3m$ when
ⓐ $m=5$
ⓑ $m=-5$

310. $18-4n$ when
ⓐ $n=3$
ⓑ $n=-3$

311. $2w^2-3w+7$ when $w=-2$

312. $3u^2-4u+5$ when $u=-3$

313. $9a-2b-8$ when $a=-6$ and $b=-3$

314. $7m-4n-2$ when $m=-4$ and $n=-9$

Translate English Phrases to Algebraic Expressions

In the following exercises, translate to an algebraic expression and simplify if possible.

315. the sum of 3 and -15, increased by 7

316. the sum of -8 and -9, increased by 23

317. the difference of 10 and -18

318. subtract 11 from -25

319. the difference of -5 and -30

320. subtract -6 from -13

321. the product of -3 and 15

322. the product of -4 and 16

323. the quotient of -60 and -20

324. the quotient of -40 and -20

325. the quotient of -6 and the sum of *a* and *b*

326. the quotient of -7 and the sum of *m* and *n*

327. the product of -10 and the difference of p and q

328. the product of -13 and the difference of c and d

Use Integers in Applications

In the following exercises, solve.

329. Temperature On January 15, the high temperature in Anaheim, California, was $84°$. That same day, the high temperature in Embarrass, Minnesota was $-12°$. What was the difference between the temperature in Anaheim and the temperature in Embarrass?

330. Temperature On January 21, the high temperature in Palm Springs, California, was $89°$, and the high temperature in Whitefield, New Hampshire was $-31°$. What was the difference between the temperature in Palm Springs and the temperature in Whitefield?

331. Football At the first down, the Chargers had the ball on their 25 yard line. On the next three downs, they lost 6 yards, gained 10 yards, and lost 8 yards. What was the yard line at the end of the fourth down?

332. Football At the first down, the Steelers had the ball on their 30 yard line. On the next three downs, they gained 9 yards, lost 14 yards, and lost 2 yards. What was the yard line at the end of the fourth down?

333. Checking Account Mayra has $124 in her checking account. She writes a check for $152. What is the new balance in her checking account?

334. Checking Account Selina has $165 in her checking account. She writes a check for $207. What is the new balance in her checking account?

335. Checking Account Diontre has a balance of $-\$38$ in his checking account. He deposits $225 to the account. What is the new balance?

336. Checking Account Reymonte has a balance of $-\$49$ in his checking account. He deposits $281 to the account. What is the new balance?

Everyday Math

337. Stock market Javier owns 300 shares of stock in one company. On Tuesday, the stock price dropped $12 per share. What was the total effect on Javier's portfolio?

338. Weight loss In the first week of a diet program, eight women lost an average of 3 pounds each. What was the total weight change for the eight women?

Writing Exercises

339. In your own words, state the rules for multiplying integers.

340. In your own words, state the rules for dividing integers.

341. Why is $-2^4 \neq (-2)^4$?

342. Why is $-4^3 = (-4)^3$?

Self Check

ⓐ *After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.*

I can...	Confidently	With some help	No-I don't get it!
multiply integers.			
divide integers.			
simplify expressions with integers.			
evaluate variable expressions with integers.			
translate English phrases to algebraic expressions.			
use integers in applications.			

ⓑ *On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?*